WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2020, held in 2021

MTMACOR02T-MATHEMATICS (CC2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Show that one of the values of $(1+i\sqrt{3})^{\frac{3}{4}}+(1-i\sqrt{3})^{\frac{3}{4}}$ is $\sqrt{3}^{\frac{3}{4}}$.

(b) Find the equation whose roots are roots of the equation $x^3+3x^2-8x+1=0$ each increased by 1.

(c) If a, b, c, d are positive real numbers, not all equal, prove that $a^5+b^5+c^5+d^5>abcd\,(a+b+c+d)$.

(d) Prove that $3^{2n}-8n-1$ is divisible by 64 for all natural numbers n.

(e) Give an example of a relation on the set of positive integers, which is reflexive and transitive but not symmetric.

(f) Show that the relation $\rho=\{(1,\ 3),(3,\ 5),(5,\ 3),(5,\ 7)\}$ on the set $A=\{1,3,5,7\}$ does not satisfy symmetry and transitivity.

(g) Determine the rank of the matrix $\begin{pmatrix}1 & -1 & 2\\ 2 & 1 & -1\\ 4 & -1 & 4\end{pmatrix}$.

(h) Find a row-reduced matrix which is row equivalent to $\begin{pmatrix}0 & 0 & 2 & 2 & 0\\ 1 & 3 & 2 & 4 & 1\\ 2 & 6 & 2 & 6 & 2\end{pmatrix}$.

(i) Use Cayley-Hamilton theorem to find A^{-1}, where $A=\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}$.

(j) Find A^{50}, where $A=\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$.

2. (a) If a, b, c, d be all positive real numbers and $s=a+b+c+d$, prove that 4

$$81\,abcd\le(s-a)(s-b)(s-c)(s-d)\le\frac{81}{256}s^4$$

(b) If α,β,γ be real numbers and $\beta+\gamma>\alpha$, $\gamma+\alpha>\beta$, $\alpha+\beta>\gamma$, show that 4

$$(\beta+\gamma-\alpha)(\gamma+\alpha-\beta)(\alpha+\beta-\gamma)\le\alpha\beta\gamma$$

3. (a) Express $z=\dfrac{-1+i\sqrt{3}}{1+i}$ in polar form and then find the modulus and argument of z. 2+2

   (b) Prove that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. 4

4. (a) Solve the equation $2x^4+5x^3-15x^2-10x+8=0$, whose roots are in geometric progression. 4

   (b) If α be a root of the cubic $x^3-3x+1=0$ then show that the other roots are (α^2-2) and $(2-\alpha-\alpha^2)$. 4

5. (a) If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\sum(\beta+\gamma-\alpha)^3$. 4

   (b) Solve the equation $x^3-15x^2-33x+847=0$. 4

6. (a) Find the equation whose roots are the roots of the equation $x^4-8x^2+8x+6=0$, each diminished by 2. 4

   (b) Solve the equation $x^4-4x^3+5x+2=0$. 4

7. (a) By the principle of mathematical induction, prove that 4

$$3^{2n+1}+(-1)^n 2 \equiv 0 \pmod 5 \text{ for all } n\in\mathbb{N}.$$

   (b) Prove that the product of any three consecutive integers is divisible by 6. 4

8. (a) Examine whether the relation ρ is an equivalence relation on the set S of all integers where 4

$$\rho=\{(a,b)\in S\times S:\ |a-b|\le 3\}$$

   (b) Show that the equivalence relation on a set S determines a partition of S. 4

9. (a) If $f:A\to B$ and $g:B\to C$ be two mappings such that $g\circ f:A\to C$ is injective, then prove that f is injective. 4

   (b) If $f:S\to T$ is one one onto, then prove that $f^{-1}:T\to S$ is one one onto. 4

10. (a) Let $\mathbb{R}$ be the set of all real numbers and $(-1, 1)$ be the interval defined by 4

$$(-1,1)=\{x\in\mathbb{R}:\ -1<x<1\}$$

   Prove that the mapping $f:\mathbb{R}\to(-1,1)$ defined by

$$f(x)=\frac{x}{1+|x|},\ \ \forall\, x\in\mathbb{R}$$

   is one to one and onto.

   (b) Suppose $f:A\to B$, $g:B\to C$ be two mappings. 2+2

   (i) If f and g are both injective, show that $g\circ f$ is also injective.

   (ii) If $g\circ f$ is injective, then show that f is injective.

11.(a) Find the values of k for which the system of equations 4

$$x+y-z=1$$
$$2x+3y+kz=3$$
$$x+ky+3z=2$$

has (i) no solution, (ii) more than one solutions, (iii) unique solution.

(b) Reduce the matrix 4

$$A=\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{pmatrix}$$

to a row-reduced Echelon form and hence find its rank.

12.(a) Use Cayley-Hamilton theorem to express A^{-1} as a polynomial in A and then compute A^{-1} where 2+2

$$A=\begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$$

(b) Show that the eigen values of a real symmetric matrix are all real. 4

13.(a) If k be a non-zero scalar, then prove that the eigen values of kA are k times the eigen values of A. 3

(b) Find the eigen values and the corresponding eigen vectors of the matrix 5

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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