WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-II Examination, 2020

MATHEMATICS

PAPER: MTMA-IV

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Full Marks- 8

1. Answer any ***four*** questions from the following: 2×4=8

(a) Find the equation of a sphere which passes through the centre of the sphere $x^2+y^2+z^2-4x+2y-4z+3=0$ and touch this sphere at (1, 1, 1). 2

(b) Find the equations of the generators of the hyperbolic paraboloid $25x^2-4y^2=z$ passing through the point (0, 1, –4). 2

(c) Find the complete integral of $q=5p^2+2$. 2

(d) Write the following game problem as a linear programming problem with respect to the player *B*. 2

Player *B*

Player *A*		B_1	B_2	B_3
	A_1	0	–1	2
	A_2	1	0	–1
	A_3	–2	1	0

(e) Check whether the set $X=\{(x,\ y):\ xy\le 4\}$ is convex. 2

(f) An engine working at 400 H.P. pulls a train of 150 tons along a level track, the resistances being 15 lbs-wt per ton. When the velocity of the train is 30 m.p.h., find its acceleration. 2

(g) If the radial and cross-radial velocities of a particle be μr and $\lambda\theta$ respectively then find the path of the particle. 2

(h) The law of motion of a particle in a straight line is $s=\frac{1}{2}vt$. Show that the acceleration is constant. 2

GROUP-B

Full Marks- 42

Answer any *three* questions from the following 14×3=42

2. (a) Reduce the equation $x^2+2y^2-3z^2-4yz+8zx-12xy+1=0$ to its canonical form and find the nature. 7

(b) Find the equation of the cone with vertex at the origin and which passes through the curves given by $x^2+y^2+z^2+x-2y+3z-6=0$ and $x^2+y^2+z^2+2x-3y+4z-7=0$. 7

3. (a) Solve: 7

$$2\frac{d^2y}{dx^2}-\frac{dy}{dx}+9y-\frac{d^2z}{dx^2}-\frac{dz}{dx}-3z=0 \text{ and } 2\frac{d^2y}{dx^2}+\frac{dy}{dx}+7y-\frac{d^2z}{dx^2}+\frac{dz}{dx}-5z=0$$

(b) Solve the dual of the problem 7

Maximize $Z=3x_1+4x_2$

Subject to $x_1-x_2\le 1$

$3x_1+x_2\ge 4$

$x_1-3x_2\le 3$

$x_1,\ x_2\ge 0$

From the dual solution, discuss the nature of the primal problem.

4. (a) Solve the following transportation problem: 7

	A	*B*	*C*	*D*	
I	6	4	2	7	8
II	5	1	4	6	14
III	6	5	2	5	9
IV	4	3	2	1	11
	7	13	12	10	

(b) Solve graphically the following game problem: 7

Player *B*

		B_1	B_2
Player *A*	A_1	1	–3
	A_2	3	5
	A_3	–1	6
	A_4	4	1
	A_5	2	2
	A_6	–5	0

5. (a) The velocity at any point of a central orbit is $\frac{1}{n}$th of what it would be for a circular orbit at the same distance; show that the central force varies as $\frac{1}{r^{2n^2+1}}$ and the equation of its orbit is $r^{n^2-1} = a^{n^2-1}\cos(n^2-1)\theta$. 7

(b) Find the tangential and normal components of velocity and acceleration of a particle which describes a plane curve. 7

6. (a) A particle hangs at rest at the end of an elastic string whose unstretched length is a. In the position of equilibrium, the length of the string is b and $\frac{2\pi}{n}$ is the time of an oscillation about this position. At time zero, when the particle is in equilibrium, the point of suspension beings to move so that the downward displacement at t is $c\sin pt$. Show that the length of the string at time t is $b - \frac{cnp}{n^2-p^2}\sin nt + \frac{cp^2}{n^2-p^2}\sin pt$. If $p = n$, show that the length of the string at time t is $b - \frac{c}{2}\sin nt - \frac{cn}{2}t\sin nt$. 7

(b) A particle is projected under gravity with a velocity u at an angle α to the horizon in a medium whose resistance equal to mk times the velocity. Show that the path of the particle is $y = \frac{g}{k^2}\log\left(1 - \frac{kx}{u\cos\alpha}\right) + \frac{x}{u\cos\alpha}\left(u\sin\alpha + \frac{g}{k}\right)$. Find the horizontal range of the particle. 7

7. (a) If a rocket, originally of mass M, throws off every unit of time a mass eM with relative velocity V, and if M' be the mass of the case etc., show that it cannot rise at once unless $eV > g$, not at all unless $e\frac{MV}{M'} > g$. If it just rise vertically at once, show that the greatest velocity is $V\log\frac{M}{M'} - \frac{g}{e}\left(1 - \frac{M'}{M}\right)$ and that the greatest height it reaches is $\frac{V^2}{2g}\log\left(\frac{M}{M'}\right)^2 + \frac{V}{e}\left(1 - \frac{M'}{M} - \log\frac{M}{M'}\right)$. 7

(b) A mass m, after falling freely through d feet beings to rise a mass M greater than itself connected with it by means of an inextensible string passing over a smooth pulley. Show that M will have returned to the original position at the end of time $\frac{2m}{M-m}\sqrt{\frac{2d}{g}}$. 7

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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