



**WEST BENGAL STATE UNIVERSITY**

B.Sc. Honours/Programme 1st Semester Examination, 2020, held in 2021

**MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)**

**DIFFERENTIAL CALCULUS**

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.  
Candidates are required to give their answers in their own words as far as practicable.  
All symbols are of usual significance.*

**Answer Question No. 1 and any five from the rest**

1. Answer any *five* questions from the following: 2×5 = 10
- (a) Evaluate  $\lim_{x \rightarrow 0} (1 + 3x)^{\frac{2}{x}}$
- (b) Find  $\lim_{x \rightarrow 2} \sqrt{x-2}$  if it exists.
- (c) Show that  $f(x) = 2x^2 + 3x + 5$  is continuous for any real number  $x$ .
- (d) Find  $\frac{dy}{dx}$  if  $(\cos x)^y = (\sin y)^x$ .
- (e) If  $y = \frac{x}{x+1}$ , show that  $y_5(0) = 5!$ .
- (f) At what point is the tangent to the parabola  $y = x^2$  parallel to the straight line  $y = 4x - 5$ .
- (g) Find the points of extremum value of the function  $f(x) = \sin x(1 + \cos x)$  in  $[0, 2\pi]$ .
- (h) If  $f(x, y) = x \log y$  then show that  $f_{xy} = f_{yx}$ .
- (i) Find the asymptotes of the curve  $x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 5 = 0$ .
- (j) Find the radius of curvature of the curve  $xy = 12$  at  $(3, 4)$ .

2. (a) A function  $f$  is defined as follows:

$$f(x) = \begin{cases} x^2 + ax & , \text{ if } 0 \leq x < 1 \\ 3 - bx^2 & , \text{ if } 1 \leq x \leq 2 \end{cases}$$

If  $\lim_{x \rightarrow 1} f(x) = 4$ , find the value of  $a$  and  $b$ .

4

- (b) If  $f, g : D \rightarrow \mathbb{R}$  are two functions such that  $\lim_{x \rightarrow c} f(x)$  and  $\lim_{x \rightarrow c} g(x)$  exists finitely, then prove that  $\lim_{x \rightarrow c} [f(x) + g(x)] = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x)$ . 4
3. (a) State and prove Lagrange's mean value theorem. 5
- (b) If  $f(x, y) = \tan^{-1} \frac{y}{x} + \sin^{-1} \frac{y}{x}$ , find the value of  $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$  at the point (1, 1). 3
4. (a) If  $y = e^{a \sin^{-1} x}$ , prove that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$ . 5
- (b) Find the radius of curvature of the cycloid  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$  at any point  $\theta$ . 3
5. (a) Show that the function  $f$  is continuous at  $x=1$  but not differentiable at  $x=1$  where
- $$f(x) = \begin{cases} x+1 & , \text{ if } 0 \leq x < 1 \\ 3-x & , \text{ if } 1 \leq x \leq 2 \end{cases}$$
- (b) Find the points on the curve  $y = 2x^3 - 15x^2 + 34x - 20$  where the tangents are parallel to the straight line  $y + 2x = 0$ . 4
6. (a) If  $f(x, y) = \begin{cases} \frac{x^3 y}{x^2 + y^2} & , \quad x^2 + y^2 \neq 0 \\ 0 & , \quad x = 0, y = 0 \end{cases}$  5
- prove that  $f_{xy}(0, 0) \neq f_{yx}(0, 0)$ .
- (b) Find the nature of double points of the curve  $(2y + x + 1)^2 = 4(1 - x)^5$ . 3
7. (a) Determine the points of discontinuities of the function 4
- $$f(x) = \begin{cases} \sin \frac{1}{x} & , \quad x \leq 0 \\ 2x & , \quad 0 < x < 1 \\ 0 & , \quad x = 1 \\ \frac{x^2 - 1}{x - 1} & , \quad 1 < x \end{cases}$$
- (b) Prove that  $\frac{x}{1+x} < \log(1+x) < x$  for all  $x > 0$ . 4
8. (a) Determine the Taylor's series expansion of  $f(x) = \cos x$ . 5
- (b) If a function  $f$  is differentiable on  $[0, 1]$  show that the equation  $f(1) - f(0) = \frac{f'(x)}{2x}$  has at least one root in  $(0, 1)$ . 3

9. (a) If  $u = f(x, y)$  and  $x = r \cos \theta$ ,  $y = r \sin \theta$  then prove that 4

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial u}{\partial \theta}\right)^2$$

(b) Show that the area of a rectangle inscribed in a circle is the maximum when it is a square. 4

10.(a) A function  $f$  is thrice differentiable on  $[a, b]$  and  $f(a) = 0 = f(b)$  and  $f'(a) = 0 = f'(b)$ . Prove that there is a number  $c$  in  $[a, b]$  such that  $f'''(c) = 0$ . 3

(b) If  $u = f(y - z, z - x, x - y)$  then show that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ . 5

11.(a) Show that the radius of curvature at any point  $(r, \theta)$  on the curve  $r = a(1 - \cos \theta)$  varies as  $\sqrt{r}$ . 5

(b) If  $f(x) = 2|x| + |x - 2|$ , find  $f'(1)$ . 3

12.(a) Find the asymptotes of the following curve: 5

$$x = \frac{t^2}{1+t^3}, \quad y = \frac{t^2+2}{1+t}$$

(b) If  $\lim_{x \rightarrow 0} \frac{\sin 2x + a \sin x}{x^3}$  be finite, find the value of 'a' and the limit. 3

13.(a) State and prove Leibnitz's theorem on successive differentiation. 5

(b) Find the radius of curvature of the curve  $y = xe^{-x}$  at its maximum point. 3

**N.B. :** Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

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