WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2020, held in 2021

PHSACOR01T-PHYSICS (CC1)

MATHEMATICAL PHYSICS I

Time Allotted: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest.

1. Answer any ***ten*** questions from the following: 2×10=20

(a) Sketch: $f(\theta)=1+\frac{1}{2}\sin^2\theta$ for $0\le\theta\le 2\pi$.

(b) Show that $f(x)=\frac{|x|}{x}$ is discontinuous at $x=0$, where $f(0)=0$.

(c) If $d\varphi(x,\, y)=M(x,\, y)\,dx+N(x,\, y)\,dy$, where φ is a well-behaved function of its arguments, then show that

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

(d) Solve: $\frac{dy}{dx}+2xy=4x$

(e) Determine a unit vector perpendicular to the plane of $\vec{A}=2\hat{i}-6\hat{j}-3\hat{k}$ and $\vec{B}=4\hat{i}+3\hat{j}-\hat{k}$.

(f) The position vector $\vec{r}$ of any arbitrary point on the surface satisfies the equation $|\vec{r}\,|=k$, a constant. Identify the geometry of the surface and justify your answer.

(g) $\vec{r}$ is the position vector of an arbitrary point in a three-dimensional space. Using Cartesian coordinate system, find gradient of $1/|\vec{r}\,|$ at any point away from the origin.

(h) For a vector field $\vec{F}(x,\, y,\, z,\, t)$ show that $dF=(d\vec{r}\cdot\vec{\nabla})\,\vec{F}+\frac{\partial\vec{F}}{\partial t}dt$.

(i) Show that $\vec{\nabla}\times\vec{r}f(r)=0$ where r is the magnitude of position vector of any arbitrary point in three-dimensional space ($f(r)$ being differentiable everywhere).

(j) ϕ is a scalar function satisfying the equation $\nabla^2\phi=0$. Show that $\vec{\nabla}\phi$ is both solenoidal and irrotational.

(k) Show that $\oiint_S d\vec{S}=0$.

(l) A dice is thrown. What is the probability that the number obtained is a prime number?

(m) There are five green and seven red balls. Two balls are selected one by one without replacement. Find the probability that the first is green and the second is red.

(n) What is meant by a probability distribution function? Cite an example.

2. (a) Prove the identity $\vec{\nabla}.(\vec{A}\times\vec{B}) = \vec{B}.(\vec{\nabla}\times\vec{A}) - \vec{A}.(\vec{\nabla}\times\vec{B})$. Hence show that $(\vec{A}\times\vec{r})$ is solenoidal if $\vec{A}$ is irrotational. 3+1

(b) If $f(x, y, z) = 0$, then show that $\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$. 4

(c) A multiple-choice test consists of 100 questions. Answer to each question has four possible options among which only one is correct. If a student answers all the questions by guessing at random, then what is the expected number of correct answers given by him? 2

3. (a) Solve: $\frac{y}{x^2} + 1 + \frac{1}{x}\frac{dy}{dx} = 0$ 2

(b) Determine the constant a so that the following vector is solenoidal: 4

$$\vec{V} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$$

(c) Calculate the mean and the variance of a binomial distribution. 2+2

4. (a) The relativistic sum w of two velocities u and v in the same direction is given by 4

$$\frac{w}{c} = \frac{\frac{u}{c} + \frac{v}{c}}{1 + \frac{uv}{c^2}}$$

If $u/c = v/c = 1-\alpha$, where $0 \le \alpha \le 1$, find w/c in powers of α. Show terms only up to α^3.

(b) Find the directional derivative of $\varphi(x, y, z) = x^2 y + xz$ at (1, 2, –1) along the direction of $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$. 3

(c) Use Green's theorem on a plane to show that the area bounded by a simple closed curve C is $\frac{1}{2}\oint_C (x\,dy - y\,dx)$. 3

5. (a) An integer N is chosen at random with $1 \le N \le 100$. What is the probability that N is a perfect square? 2

(b) Obtain the complementary function of the differential equation 3

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = \cos 2x$$

(c) Evaluate the line integral of $\vec{A}(x, y, z) = x^2\hat{i} + y^2\hat{j} - z^2\hat{k}$, from the origin to (a, b, c), along the path given parametrically by $x = at^2,\ y = bt,\ z = c\sin(\pi t/2)$. Does the result depend on the path? Justify your answer. 3+2

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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