# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2020, held in 2021

## MTMACOR01T-MATHEMATICS (CC1)

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

### Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Evaluate the limit: $\lim\limits_{x\to(\frac{\pi}{2})+}(\tan x)^{2x-\pi}$

(b) If $y=e^{m\sin^{-1}x}$, show that $(1-x^2)\,y_{n+2}-(2n+1)x\,y_{n+1}-(n^2+m^2)\,y_n=0$. Also find $y_n(0)$.

(c) Find the interval where the curve $y=e^x(\cos x+\sin x)$ is concave upwards or downwards for $0<x<2\pi$.

(d) Find the vertical and horizontal asymptotes of the following curve:

$$f(x)=\begin{cases}\dfrac{(x+1)^2}{x^2+4x+3} & ; \text{ if } x\neq -1 \text{ or } -3\\ 0 & ; \text{ otherwise}\end{cases}$$

(e) A sphere of radius *k* passes through the origin and meets the axes in *A*, *B*, *C*. If $(\alpha,\beta,\gamma)$ be the centroid of the triangle *ABC*, then find the value of $\alpha^2+\beta^2+\gamma^2$.

(f) Examine the curve $x=6t^2$, $y=4t^3-3t$ for concavity and convexity.

(g) Find the arc length of the curve $y=\dfrac{e^x+e^{-x}}{2}$, $0\le x\le 2$.

(h) Find the equation of the generating lines of the hyperboloid $yz+2zx+3xy+6=0$ which pass through the point (–1, 0, 3).

(i) Solve: $(4x^2y-6)\,dx+x^3dy=0$

(j) Test whether the equation $x\,dx+y\,dy+\dfrac{x\,dy-y\,dx}{x^2+y^2}=0$ is exact or not.

2. (a) Find the point of inflexion, if any of the curve $x=(\log y)^3$. 4

(b) Trace the curve $x^3+y^3=3axy$. 4

3. (a) Prove that the envelope of circle whose centres lie on the rectangular hyperbola $xy=c^2$ and which pass through its centre is $(x^2+y^2)^2=16c^2xy$. 4

(b) Find the asymptotes of the curve $x^2(x+y)(x-y)^2+2x^3(x-y)-4y^3=0$. 4

4. (a) Assuming evolute as the envelope of normals find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 4

(b) Find the value of *a*, such that $\lim_{x\to 0}\frac{a\sin x-\sin 2x}{\tan^3 x}$ is finite. Find the limit. 4

5. (a) If $I_{m,n}=\int\cos^m x\cos nx\,dx$ then prove that, 4

$$I_{m,n}=\frac{\cos^m x\sin nx}{m+n}+\frac{m}{m+n}\,I_{m-1,n-1}$$

(b) Find the surface area formed by the revolution of $x^2+4y^2=16$ about the *x*-axis. 4

6. (a) Derive the reduction formula for $\int\sec^n x\;dx$ and hence evaluate $\int\sec^7 x\;dx$. 4

(b) Show that the length of the parabola $y^2=4ax$ cut-off by its latus-rectum is $2a\left[\sqrt{2}+\log\left(1+\sqrt{2}\right)\right]$. 4

7. (a) Discuss the nature of the conic $x^2+4xy+y^2-2x+2y+a=0$ for different values of '*a*'. 4

(b) Determine the nature of the conic $r=\frac{1}{4-5\cos\theta}$. Find the eccentricity, the length of the latus rectum and directrices. 4

8. (a) Show that if a right circular cone has three mutually perpendicular generators, the semivertical angle is $\tan^{-1}\sqrt{2}$. 4

(b) Prove that the central sections of the conicoid $(a-b)x^2+ay^2+(a+b)z^2=1$ are at right angles and that the umbilics are given by $x=\pm\sqrt{\frac{a+b}{2a(a-b)}}$ , $y=0$, $z=\pm\sqrt{\frac{a-b}{2a(a+b)}}$. 4

9. (a) Prove that the centres of spheres which touch the straight lines $y = mx,\ z = c$ and $y = -mx,\ z = -c$ lie on the surface $mxy + cz(1+m^2) = 0$. 4

(b) Find the equation of the cylinder whose generating line is parallel to the *z*-axis and the guiding curve is $x^2 + y^2 = z$ , $x + y + z = 1$. 4

10.(a) Solve: $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$ 4

(b) Solve: $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$ 4

11.(a) Show that the equation of the curve, whose slope at any point (*x*, *y*) is equal to $xy\,(x^2y^2 - 1)$ and which passes through the point (0, 1) is $x^2y^2 = 1 - y^2$. 4

(b) Solve: $\sec^2 y\,\frac{dy}{dx} + 2x\tan y = x^3$ 4

12.(a) Prove that $(x + y + 1)^{-4}$ is an integrating factor of the equation $(2xy - y^2 - y)\,dx + (2xy - x^2 - x)\,dy = 0$ and hence solve it. 4

(b) Show that the differential equation of the circles through the intersection of the circle $x^2 + y^2 = 1$ and the line $x - y = 0$ is given by 4

$$(x^2 - 2xy - y^2 + 1)\,dx + (x^2 + 2xy - y^2 - 1)\,dy = 0$$

13.(a) Find the surface area of the reel formed by the revolution of cycloid $x = a\,(\theta + \sin\theta),\ y = a\,(1 - \cos\theta)$ about the tangent at the vertex. 3

(b) If $I_n = \int x^n \cos x\,dx$, then prove that 3

$$I_n = x^n \sin x + nx^{n-1}\cos x - n(n-1)\,I_{n-2}$$

use this to determine $\int x^5 \cos x\,dx$.

(c) Find the singular solution of $9\left(\frac{dy}{dx}\right)^2 (2 - y)^2 = 4(3 - y)$. 2

**N.B. :** *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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